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TITLE: NODE-BASED NETWORK ANALYSIS AND ITS APPLICATION TO MINE VENTILATION

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NODE-BASED NETWORK ANALYSIS AND ITS APPLICATION TO MINE VENTILATION

by

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ABSTRACT

A numerical procedure to model flow networks using a node-based approach is discussed, and the analysis encompasses both steady-state and transient solutions. A family of computer codes has been developed using the node-based approach. This technique is illustrated by applying two of the computer codes to mine ventilation networks.

INTRODUCTION

A ventilation network analysis method based on successive numerical corrections at nodes is introduced. Most mine ventilation network computer codes use successive numerical corrections of the flows in a network to achieve a balanced. These two approaches first were put forth by Hardy Cross many years ago (Cross, 1936). The flow correction method was used initially because there are fewer flow simultaneous equations than nodal equations, and this was very beneficial when balancing a network by hand calculations. However, using a computer code to balance the pressures in the network overcomes this apparent disadvantage to the nodal approach.

S. S. Bhamidipati and J. A. Procarione discuss a linear analysis of flow networks based on solving nodal head equations (Bhamidipati and Procarione, 1986). They use a matrix approach to solve the nodal set of equations and report that this formulation offers significant advantages over traditional network analyses.

The numerical procedure described here uses an iterative solution method and produces both steady-state and transient solutions. In addition, both incompressible and compressible flows can be taken into account. Flows, pressures, temperatures, densities, and material concentrations can be calculated.

The mathematics and numerical procedures used when temperature and density variations are unimportant are given. When these effects are important, the basic analysis is extended by adding momentum and energy equations to obtain the solution. These analyses have resulted in several computer codes that are easily understood by a ventilation engineer. Several classical mine ventilation problems will be used to illustrate the approach.

NUMERICAL SOLUTION PROCEDURE

Temperature and Density Variations Unimportant

The volume flow rate, Q_k, is related to the pressure drop ($P_i - P_j$) for a duct between two nodal locations as follows.

$$Q_k = B_k \, (P_i - P_j)^M \quad , \tag{1}$$

where

B_k = resistance of branch k,
M = constant, and
P_i, P_j = nodal pressures at ends of branch k.

Applying the basic nodal relationship of conservation of flow at the nodes requires

$$\Sigma Q_k = 0 \quad . \tag{2}$$

If a node contains a certain volume or capacitance, the following relationship applies.

$$\frac{dp}{dt} = \frac{\rho RT}{V} \, (Q_{in} - Q_{out}) \quad , \tag{3}$$

where

ρ = density,
R = gas constant,
T = temperature,
V = node volume,
t = time, and
Q_{in}, Q_{out} = branch flows in and out of the volume.

If the variations of ρ and T are small or unimportant, they can be treated as constants in the analysis.

MASTER

Using a perturbation technique, we note that for a small perturbation ($\delta\tilde{P}$), the correct value for pressure P_j is

$$P_j = \tilde{P}_j + \delta\tilde{P} \quad , \tag{4}$$

where the sign ~ indicates a temporary value. Returning to Eq. (1) and substituting Eq. (4) gives

$$Q_k = B_k\,(P_i - P_j - \delta\tilde{P})^N \quad . \tag{5}$$

Using a Taylor series expansion of Eq. (5) gives a linear relationship between flow and pressure as

$$Q_k = \tilde{A}_k - \tilde{C}_k\,\delta\tilde{P} \quad , \tag{6}$$

where A_k and C_k are temporary iterative values based on previous values of pressure. Equation (6) can be substituted into Eq. (2) to yield

$$\delta\tilde{P}_j = \frac{\Sigma\tilde{A}_k}{\Sigma\tilde{C}_k} \quad . \tag{7}$$

This is essentially a formulation of Newton's method. An implicit iterative numerical scheme is used to solve Eq. (7) for the pressure correction at each node. The iterative process continues until the pressure correction $\delta\tilde{P}$ approaches zero or until a convergence criterion is met. The numerical scheme is altered slightly for the equation of state that was formulated using Eq. (3) for capacitance nodes.

Temperature/Density Variations Important

For cases in which temperature and density changes are severe, Eq. (1) is not sufficient because the flow-pressure relationship is more complicated. In particular, the density variation must be considered simultaneously with the pressure variation. The approach taken for these cases is outlined in a Los Alamos National Laboratory report (Gregory et al., 1979).

Because Eqs. (2) and (3) are not adequate to describe the conservation of mass, the energy equation must be added to describe the flow dynamics. Thus, we must use a numerical scheme to solve a set of nonlinear algebraic equations with two unknowns.

The conservation of mass can be developed as follows.

$$\sum_k q_k \rho_k Q_k = \sum_k q_k \dot{m}_k = 0 \quad , \tag{8}$$

where $\dot{m}_k$ and Q_k are the mass and volumetric flow rates, respectively, in branch k. ρ_k is the density in branch k, and q_k is used to adjust for the proper flow direction; $q_k = +1$ for the downstream node of a branch and $q_k = -1$ for the upstream node Equation (8) can be made more general to allow for mass accumulation at the node by the following relationship [analogous to Eq. (3)]

$$\frac{d\rho}{dt} = \frac{1}{V}\left[\sum_k q_k \dot{m}_k + \dot{M}_s\right] \quad , \tag{9}$$

where $\dot{M}_s$ is an arbitrary mass source per unit time for the volume, V, of the node.

The second relationship required in this analysis is the energy equation with its corresponding effect on both nodal temperatures and pressures. The energy equation used in the analysis is

$$\frac{dp}{dt} = \frac{R\gamma}{V}\left[\sum_k q_k \dot{m}_k \left(T_k + \frac{vk^2}{2C_p}\right) + \dot{M}_s T_s + \frac{\dot{E}_s}{C_p}\right] \quad , \tag{10}$$

where

- v = branch velocity,
- R = gas constant,
- γ = specific heat ratio,
- C_p = specific heat at constant value,
- T_k = branch temperature,
- T_s = source temperature, and
- $\dot{E}_s$ = energy source.

This equation is developed in detail in a report by Tang (Tang et al., 1981).

In this analysis, we require a more complex relationship between the pressure and flow than that used in Eq. (1). Using Fig. 1, the momentum equation for branch flow $\dot{m}$ is

$$I\,\frac{d\dot{m}}{dt} = (P_i - P_j) - \frac{sK_{eff}\,\dot{m}^2}{2A^2\rho} \quad , \tag{11}$$

where

I = branch inertia,

$$= \frac{\ell_i}{2A_i} + \frac{\ell}{A} + \frac{\ell_j}{2A_j} \quad , \tag{12}$$

$$K_{eff} = \left(\frac{f\ell_i}{2D_i} + K_i\right)\left(\frac{A}{A_i}\right)^2 + \frac{f\ell}{D} + K^2 + \left(\frac{f\ell_j}{2D_j} + K_j\right)\left(\frac{A}{A_j}\right)^2 \quad , \tag{13}$$

$$s = +1 \text{ if } \dot{m} \geq 0 \quad , \text{ and} \tag{14a}$$

$$s = -1 \text{ if } \dot{m} \leq 0 \quad . \tag{14b}$$

The duct flow equation [Eq. (11)] is derived in more detail in Tang (Tang, 1982). The true upstream node density and the pressure differential

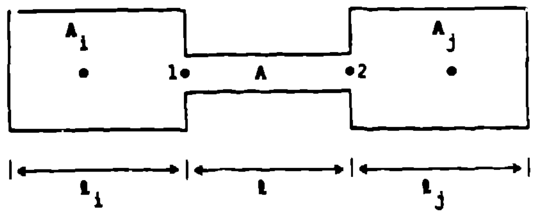

FIGURE 1. Branch with sudden area change.

are $(P_1 - P_2)$, not $(P_i - P_j)$, so when the flow is positive from i to j,

$$P_2 = P_j \quad . \tag{15}$$

However, P_1 and P_i are connected through the isentropic relation

$$\frac{P_1}{P_i} = r_p = \left(\frac{T_1}{T_i}\right)^{\frac{\gamma}{\gamma-1}} \quad , \tag{16}$$

with

$$T_i + \frac{v_i^2}{2C_p} = T_1 + \frac{v_1^2}{2C_p} \quad . \tag{17}$$

For the density relationship, we have

$$\frac{\rho_1}{\rho_i} = r_\rho = \left(\frac{T_1}{T_i}\right)^{\frac{1}{\gamma-1}} \quad . \tag{18}$$

Therefore, the continuity between i and 1,

$$\dot{m} = \rho_i v_i A_i = \rho_1 v_1 A_1 \quad , \tag{19}$$

can be solved to obtain r_ρ for a given $\dot{m}$ and node condition.

We can cast the flow equation into a form similar to Eq. (6) by adding both the perturbation pressure and density.

$$\dot{m} = \bar{A} - \bar{C}\delta\bar{P} - \bar{E}\delta\bar{\rho} \quad , \tag{20}$$

where $\bar{A}$, $\bar{C}$, and $\delta\bar{\rho}$ have the same meaning as in Eq. (6) and $\bar{E}$ is a temporary constant associated with the density perturbation term,

$$\rho_i = \bar{\rho}_i + \delta\bar{\rho}_i \quad . \tag{21}$$

The values for $\bar{A}$, $\bar{C}$, and $\bar{E}$ in finite difference form are given in Eqs. (22)–(24).

$$a\bar{A}^2 + \frac{1}{\Delta t}\bar{\rho}_1\bar{A} + \bar{\rho}_1\left[(P_j - \bar{P}_1) - \frac{1}{\Delta t}\dot{m}_o\right] = 0 \quad , \tag{22}$$

$$\bar{C} = \frac{\bar{\rho}_1 \bar{r}_\rho}{2a\bar{A} + \frac{1}{\Delta t}\bar{\rho}} \quad . \tag{23}$$

$$\bar{E} = \bar{r}_\rho\left\{\frac{1}{\Delta t}\bar{A} + \left[(P_j - \bar{P}_i) - \frac{1}{\Delta t}\dot{m}_o\right] / \left(2a\bar{A} + \frac{1}{\Delta t}\bar{\rho}_1\right)\right\} , \tag{24}$$

where

$$a = \frac{sK_{eff}}{2A^2} \quad . \tag{25}$$

Turning to the mass conservation equation [Eq. (9)], its finite-difference form is

$$\frac{V}{\Delta t}(\rho_i - \rho_{i_o}) = \Sigma q_k \dot{m}_k + \dot{M}_s \quad , \tag{26}$$

with ρ_{i_o} being the node density at the previous time. We put the variations of ρ_i and $\dot{m}_k$ into Eq. (26) and obtained

$$a_{11}\delta\bar{P}_i + a_{12}\delta\bar{\rho}_i = b_1 \quad , \tag{27}$$

where

$$a_{11} = \Sigma q_k \bar{C}_k \quad , \tag{28a}$$

$$a_{12} = \frac{V}{\Delta t} + \Sigma q_k \bar{E}_k \quad , \text{ and} \tag{28b}$$

$$b_1 = \Sigma q_k \bar{A}_k + \dot{M}_s + \frac{V}{\Delta t}(\rho_{i_o} - \bar{\rho}_i) \quad . \tag{28c}$$

Equation (27) contains two unknowns, $\delta\bar{P}_i$ and $\delta\bar{\rho}_i$. We need the energy equation to complete the system.

Using the energy equation and the concept presented above with much algebraic manipulation yields

$$a_{21}\delta\bar{P}_i + a_{22}\delta\bar{\rho}_i = b_2 \quad , \tag{29}$$

where

$$a_{21} = \frac{V}{\Delta t} + R\gamma\, \Sigma q_k C_{e_k} \quad , \tag{30a}$$

$$a_{22} = R\gamma\, \Sigma q_k E_{e_k} \quad , \text{ and} \tag{30b}$$

$$b_2 = \frac{V}{\Delta t}(P_{i_o} - \bar{P}_i) + R\gamma \left[\Sigma q_k F_{e_k} + \dot{M}_s T_s + \frac{\dot{E}_s}{C_p} \right] . \tag{30C}$$

The undefined variables in the above equations are as follows.

$$F_e = \bar{F}_e - \bar{C}_e \delta P_i - \bar{E}_e \delta\tilde{\rho}_i = \text{energy flux} , \tag{31}$$

where

$$\bar{F}_e = \bar{A}(T + \bar{R}_e) , \tag{32}$$

$$\bar{C}_e = \bar{C}(T + \bar{R}_e) + \bar{A}T\left(\frac{C_T}{\bar{r}_T} - \frac{1}{P_i}\right) + 2\bar{R}_e\left(\bar{C} - \frac{A}{\gamma-1}\frac{C_T}{\bar{r}T}\right) , \tag{33}$$

$$\bar{E}_e = \bar{E}(T + \bar{R}_e) + \bar{A}T\left(\frac{E_T}{\bar{r}_T} - \frac{1}{\bar{\rho}_i}\right) + 2\bar{R}_e\left(\bar{E} - \frac{A}{\gamma-1}\frac{\bar{E}_T}{\bar{r}_T} + \frac{\bar{A}}{\bar{\rho}_i}\right) , \tag{34}$$

$$\bar{R}_e = \frac{1}{2A^2 C_p}\left(\frac{\bar{A}}{\bar{r}_\rho \tilde{\rho}_i}\right)^2 , \tag{35}$$

$$r_\rho = \bar{r}_\rho - \frac{C_T}{\gamma-1}\left(\frac{\bar{r}_\rho}{r_T}\right)\delta\tilde{\rho}_i - \frac{\bar{E}_T}{\gamma-1}\left(\frac{\bar{r}_\rho}{\bar{r}_T}\right)\delta\tilde{\rho}_i , \tag{36}$$

$$E_T = \frac{1}{D_T}\left(\frac{2\bar{E}}{\bar{A}} + \frac{1}{\bar{\rho}_i}\right) , \tag{37}$$

$$C_T = \frac{1}{D_T}\left(\frac{2\bar{C}}{\bar{A}} + \frac{1}{\bar{\rho}_i}\right) , \tag{38}$$

$$D_T = \frac{1}{\bar{r}_T}\left\{\left(\frac{2}{\gamma-1}\right)\bar{r}_T - \left(\frac{2}{\gamma-1}\right)\left[\bar{r}_T - \left(\frac{2}{\gamma-1}\right) - \left(\frac{A_1}{A_i}\right)^2\right]^{-1} - \frac{\bar{r}_T}{1 - r_T}\right. , \tag{39}$$

$$r_T = \bar{r}_T - C_T\,\delta P_i - E_T\delta\tilde{\rho}_i , \text{ and} \tag{40}$$

$$r_T = \frac{T}{T_i} \tag{41}$$

Combining Eqs. (27) and (29) gives a solution for δP_i and $\delta\tilde{\rho}_i$:

$$\delta P_i = (b_1 a_{22} - b_2 a_{12})/(a_{11}a_{22} - a_{12}a_{21}) \text{ and} \tag{42}$$

$$\delta\tilde{\rho}_i = (a_{12}b_2 - a_{21}b_1)/(a_{11}a_{22} - a_{12}a_{21}) . \tag{43}$$

The correction terms δP_i and $\delta\tilde{\rho}_i$ are used in the numerical scheme explained above for the constant temperature and density case.

COMPUTER CODES

The numerical solution procedure we developed has led to the development of a family of computer codes.

TVENT - A computer code that can determine the pressures and flows in an arbitrary ventilation network for steady-state and mild transient conditions (Duerre et al., 1978).

EVENT - A computer code that can determine the pressures, flows, densities, and temperatures in arbitrary ventilation networks for steady-state and severe transient conditions (Tang et al., 1983).

TORAC - The same capabilities as TVENT, but it includes the capability for transport of aerosols (Andrae et al., 1985).

EXPAC - The same capabilities as EVENT, but it includes the capability for transport of aerosols (Nichols et al., 1987).

FIRAC - A computer code that can determine the pressures, flows, densities, temperatures, and heat transfer in an arbitrary ventilation network. Fire propagation and smoke transport are included (Nichols et al., 1986).

All of the computer codes are based on the nodal concept outlined above. The airflow pathways can include multiple interconnected rooms, dampers, filters, and blowers. The transient portions of the codes are used to simulate the effects of accidents such as fires and explosions or equipment failures such as loss of a blower. The codes are oriented toward building ventilation systems, but there is no reason why they cannot be applied to mine ventilation networks, which are in some ways much simpler. Applications to mine ventilation networks are presented below.

APPLICATIONS TO MINE VENTILATION NETWORKS

In this section, we discuss the application of two of the codes outlined above to selected mine ventilation networks. TORAC and EXPAC will be used to model the ventilation networks for both steady-state and transient solutions.

The first application is the classic mine ventilation network shown in Fig. 2. This is the same network analyzed by Bhamidipati and Procarione (1986). In our analysis, we have chosen to use TORAC to obtain the steady-state solution. Equations (1)--(7) are solved in an iterative, implicit manner with a convergence criterion of 0.001. We have added an extra branch to separate the fan from the flow resistance between nodes 1 and 5. In addition, we have given the fan a relatively flat blower curve. Using the resistance values shown in Fig. 2 and an arbitrary flow distribution, the steady-state results are given in Tables 1 and 2.

The second mine ventilation problem involves using the EXPAC code to model the flow network shown in Fig. 3. Figure 3 depicts a more realistic, although small, mine ventilation system. The steady-state flows and pressures at selected branches and nodes are given in Tables 3 and 4. In addition to solving for the system steady-state flows and pressures, we wish to illustrate the transient capabilities of the codes.

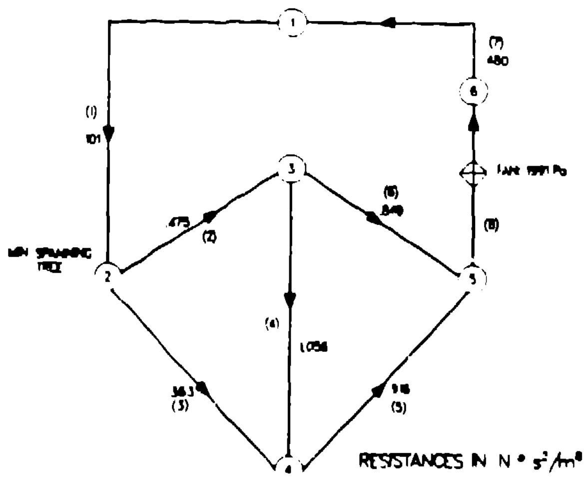

FIGURE 2. Simple network to illustrate application of node-based network analysis.

TABLE 1. TORAC Steady-State Solution of Fig. 2 Branch Flows.

Branch	Flow (cfm)	Flow (m^3/s)
1	99 473	46.94
2	46 624	22.00
3	52 893	24.96
4	-3 935	-18.57
5	48 955	23.10
6	50 663	23.91
7	99 423	46.92
8	99 423	46.92

TABLE 2. TORAC Steady-State Solution of Fig. 2 Nodal Pressures.

Node	Pressure (in. w.g.)	Pressure (Pa)
1	0.1625	40.48
2	0.7310	182.10
3	1.6549	412.24
4	1.6403	408.60
5	3.6031	897.53
6	4.4031	1096.81

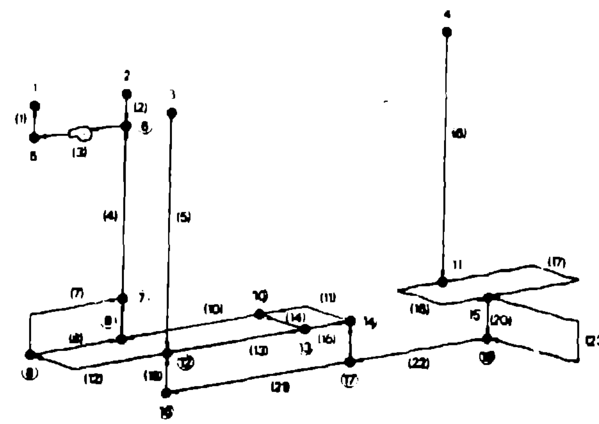

FIGURE 3. Small mine ventilation network to illustrate application of steady-state and transient analyses.

TABLE 3. Figure 3 Steady-State Branch Flow Rates.

Branch	Flow Rate (m^3/s)
6	14.16
20	6.75
23	22.22
22	28.75
4	132.15

TABLE 4. Figure 3 Steady-State Node Pressures.

Node	Pressure (kPa)
4	0.00
15	2.069
18	4.494
17	2.226
6	3.088

To illustrate the transient capabilities of the EXPAC code, we have assumed that 20 lb of TNT explodes at node 18. As shown in Figs. 4 and 5, a very strong transient is propagated through the system. Figure 4 shows the change in pressure at node 18, and Fig. 5 shows the flow rates in branches 20 and 22. Notice the stong flow reversal in branch 20 and the increase in flow for branch 22.

SUMMARY

We have developed node-based solution algorithms suitable for arbitrary flow networks. These solution algorithms can be used to obtain steady-state and transient solutions. Two solution forms are presented-- one for cases where temperature and density variations are unimportant and one for cases where they are important.

Two computer codes, TORAC and EXPAC, were selected to illustrate the application of the techniques to mine ventilation problems. The two codes are from a family of computer codes developed at Los Alamos to perform accident analyses in nuclear facilities.

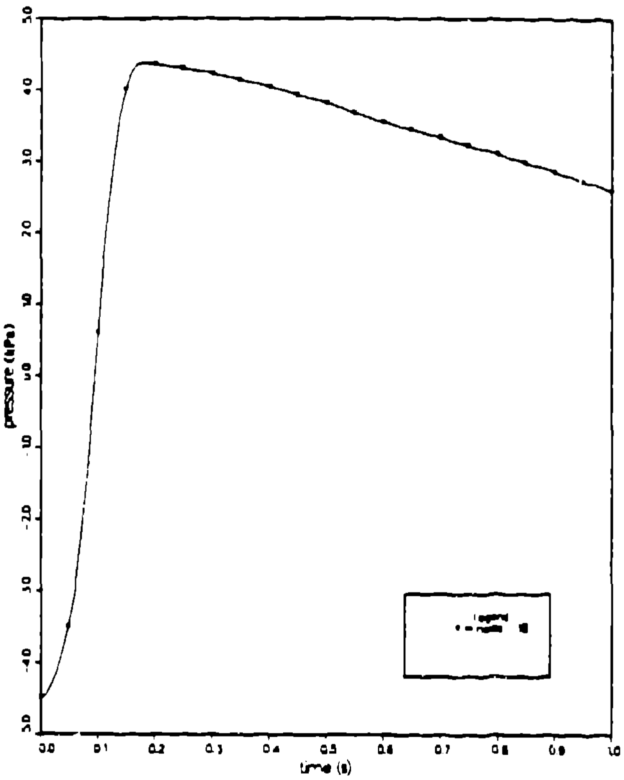

FIGURE 4. Time history of pressure at node 18 because of a simulated 20-lb TNT explosion.

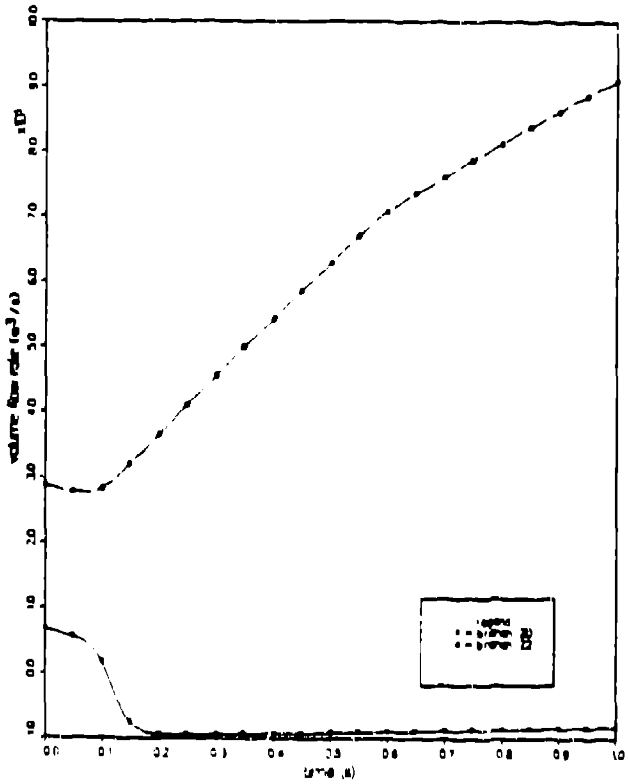

FIGURE 5. Time history of volumetric flow in branches 20 and 22 because of a simulated 20-lb TNT explosion at node 18.

Two mine schematics were analyzed. A classical mine schematic (Fig. 2) was analyzed for steady-state flows using TORAC. EXPAC was used to analyze a more complicated mine ventilation system for both steady-state and transient flows.

We have presented a solution technique and computer codes that can be used easily by the mine ventilation community for both mine design and analysis of potential accident situations involving fires and explosions and the associated aerosol movement.

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